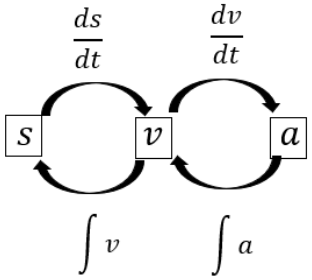
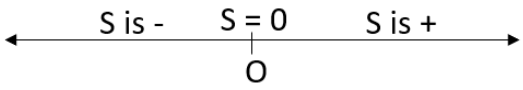
Mathematics Methods
Unit 3

Rectilinear motion

1.	**Rectilinear motion**
	Summary:  Where, s: displacement v: velocity a: acceleration
	(a) Displacement
	Displacement (S) of a particle: The distance measured from a fixed point, O, in a specific direction In layman's term, displacement is how far the particle is from the starting point.  *Displacement is represent by S. Formula: $$S = \int_b^a v \, dt$$
	Example 1: The velocity of a moving particle is equated by $v = 2t^3 + 3t^2 - 2t$. Find the displacement of particle after 5 s.

Example 2:
A particle accelerates at $5\ ms^{-2}$ which is constant along the s-axis. It has a velocity of $-1\ ms^{-2}$ at the start of the journey. Find the displacement of the particle within the range of time, $2 \leqslant t \leqslant 4$.

(b) Distance

Distance travelled by a particle: Total length of the path travelled by a particle from its original position to another position.

In layman's term, distance travelled is how far the particle has travelled.

Formula:

$$S = \int_b^a |v|\ dt$$

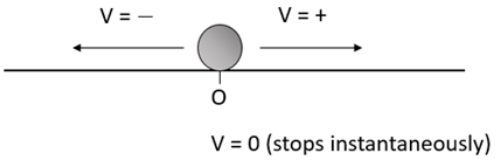
Tips:
Sketch the graph of velocity function.

Example 1:
The velocity of a moving particle is equated by $v = 2t^3 + 3t^2 - 2t$. Find the total distance travelled by particle from its origin point till $t = 1s$.

Example 2:
A particle accelerates at $5\ ms^{-2}$ which is constant along the s-axis. It has a velocity of $-1\ ms^{-2}$ at the start of the journey. Find the distance travelled by the particle from the origin to when $t = 2\ s$.

(c) Velocity

Velocity (v) is the rate of change of displacement.



V = 0 (stops instantaneously)

Formula:

$$v = \frac{ds}{dt}$$

$$v = \int a\,dt$$

Example 1:
The velocity function of a moving particle is given by $v(t) = e^{sin\,2t}$, find the velocity of the particle at $t = 2\ s$. Do not evaluate your answer.

Example 2:
A moving particle travells along a straight line, passing through a fixed point at velocity 12 ms^{-1}. Its acceleration, $a\ ms^{-2}$ is given by $a = 4t^5 + 9$ where t is the time in seconds after passing through O. Find the velocity of the particle when $t = 5$.

(d) Acceleration

Acceleration is defined as the rate of change of velocity.

Formula:

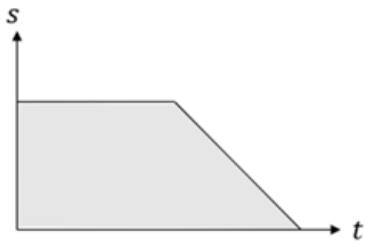
$$a = \frac{dv}{dt} \quad \text{or} \quad a = \frac{d^2s}{dt^2}$$

Example 1:
A moving particle travells along a straight path and passes through a fixed point. Its velocity, $v\ ms^{-1}$ is given by $v = 3t^2 + 6t$. Determine the acceleration of the particle when $t = 2$.

Example 2:
The velocity of a moving particle can be equated by $v(t) = cos\left(\frac{\pi t}{3}\right)$. Find the acceleration of the particle when $t = 3\ s$.

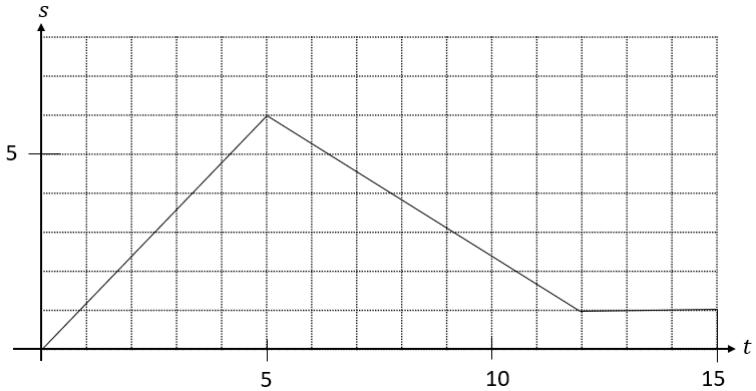
2. Graphs of rectilinear motion

(a) Displacement-time graph



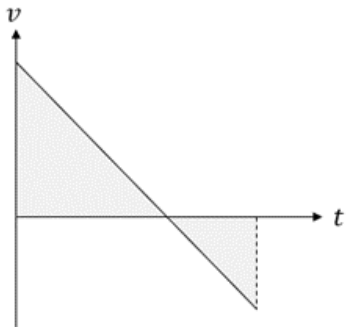
Gradient of graph: velocity, ms^{-1}
Area under graph: -

Example:



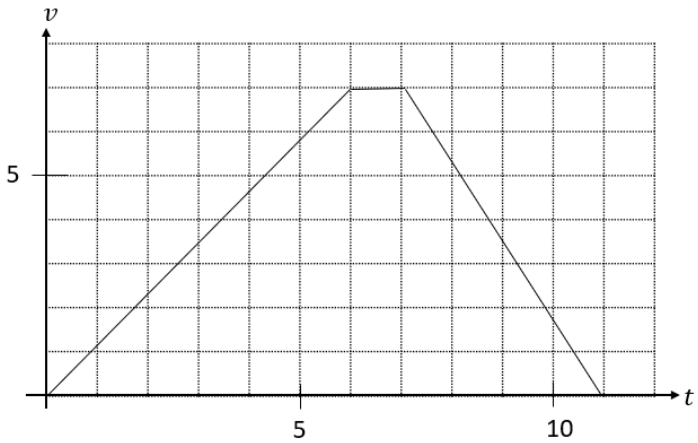
Given the displacement time graph above, find the velocity function for time $5 \leq t \leq 12$.
Hence, find the acceleration from $t = 6\ s$ to $t = 10\ s$.

(b) Velocity-time graph



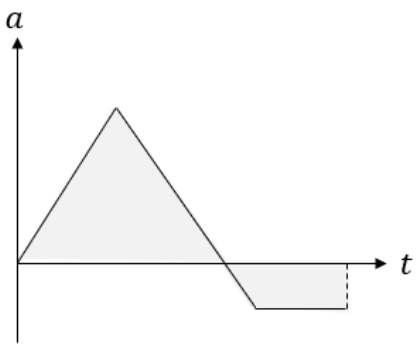
Gradient of graph: acceleration, ms^{-2}
Area under graph: displacement, m

Example:



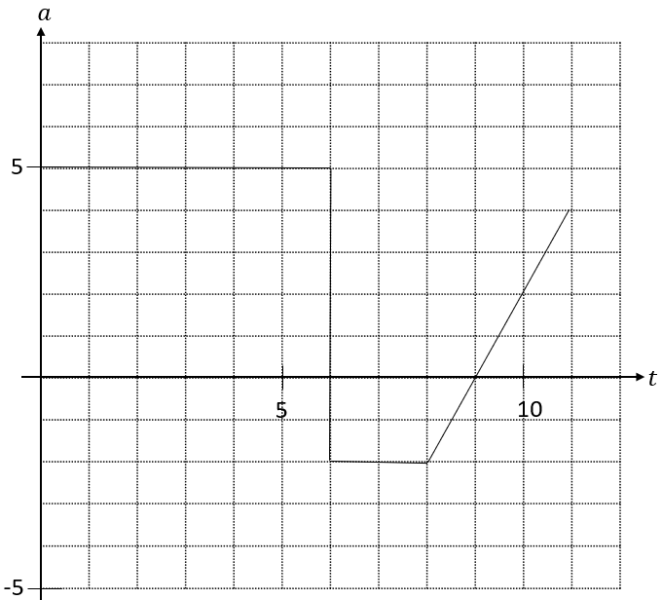
Given the velocity-time graph of a moving particle above, what is the time when particle was $28\ m$ away from its' origin?

(c) Acceleration-time graph



Gradient of graph: jerk, ms^{-3}
Area under graph: velocity, v

Example:



A person observes the motion of the particle at the 9th s of the particle's journey from its origin. Find the velocity of the particle after 2 s from the time when the person observes the motion of particle.

END